Electroweak precision constraints on the Lee-Wick standard model

Ezequiel Álvarez and Carlos Schat

CONICET and Departamento de Física, FCEyN, Universidad de Buenos Aires,
Ciudad Universitaria, Pab.1, (1428) Buenos Aires, Argentina
E-mail: sequi@df.uba.ar, schat@df.uba.ar

Leandro Da Rold

Instituto de Física, Universidade de São Paulo,
Rua do Matão 187, São Paulo, SP 05508-900, Brazil
E-mail: daroldl@fma.if.usp.br

Alejandro Szynkman

Physique des Particules, Université de Montréal,
C.P. 6128, succ. centre-ville, Montréal, QC, H3C 3J7 Canada
E-mail: szynkman@lps.umontreal.ca

ABSTRACT: We perform an analysis of the electroweak precision observables in the Lee-Wick Standard Model. The most stringent restrictions come from the S and T parameters that receive important tree level and one loop contributions. In general the model predicts a large positive S and a negative T. To reproduce the electroweak data, if all the Lee-Wick masses are of the same order, the Lee-Wick scale is of order 5 TeV. We show that it is possible to find some regions in the parameter space with a fermionic state as light as $2.4 - 3.5$ TeV, at the price of rising all the other masses to be larger than $5 - 8$ TeV. To obtain a light Higgs with such heavy resonances a fine-tuning of order a few per cent, at least, is needed. We also propose a simple extension of the model including a fourth generation of Standard Model fermions with their Lee-Wick partners. We show that in this case it is possible to pass the electroweak constraints with Lee-Wick fermionic masses of order $0.4 - 1.5$ TeV and Lee-Wick gauge masses of order 3 TeV.

KEYWORDS: Beyond Standard Model, Quark Masses and SM Parameters, Higgs Physics, Standard Model.

Contents

1. Introduction

The Standard Model (SM) describes the electroweak (EW) interactions with an incredible precision. However, the instability of the Higgs potential under radiative corrections signals our ignorance over the real mechanism of electroweak symmetry breaking (EWSB) and has lead to many extensions beyond the SM. The upcoming LHC era is likely to provide us the tools to check some of the proposed solutions to this problem.

Recently, Grinstein, O'Connell and Wise proposed a new extension of the SM [1], based on the ideas of Lee and Wick [2, 3] for a finite theory of quantum electrodynamics. The building block of the Lee-Wick proposal is to consider that the Pauli-Villars regulator describes a physical degree of freedom. In the Lee-Wick Standard Model (LWSM), this idea is extended to all the SM in such a way that the theory is free from quadratic divergences and the hierarchy problem is solved. Every SM field has a LW-partner with an associated LW-mass, these masses are the only new parameters in the minimal LWSM. A potential problem in this model is that the LW-states violate causality at the microscopic level due

to the opposite sign of their propagators. However the authors of ref. [1] argued that there is no causality violation on a macroscopic scale provided that the LW-particles are heavy and decay to SM-particles. The LWSM can be thought as an effective theory coming from a higher derivative theory. However, to insure perturbative unitarity, higher dimension operators cannot be of any type, only those compatible with a LW effective Lagrangian are acceptable [4]. In ref. [1] it was shown with a specific example that unitarity is preserved due to the unusual sign of the LW-particles width. Further considerations on the unitarity of the theory have been presented extensively in the previous literature [5 – 9], the non-perturbative formulation has been discussed in [10 – 14] and the one-loop renormalization of LW-gauge theories has been discussed in [15].

Recent work discussed the suppression of flavor changing neutral currents [16], gravitational LW particles [17] and the possibility of coupling the effective theory to heavy particles [18]. On the phenomenological side, the implications for LHC [19, 20] and ILC [21] have also been discussed.

The LWSM does not provide any information on the origin of the LW-masses. However, in order to solve the hierarchy problem these masses should not be heavier than a few TeV. On the other hand, the LW particles can not be too light without getting in conflict with EW precision observables [22]. Therefore the aim of our work is to carry out an analysis of the electroweak precision tests (EWPT) and derive bounds on the masses of the LWSM. Since the main motivation to introduce the LWSM was to solve the hierarchy problem, large LW-masses will be a source of fine-tuning and will partially spoil the original motivation. In this way the success of the LWSM is associated to its efficiency to pass the EWPT without introducing a severe fine-tuning in the theory.

On the experimental side, determining the parameter space allowed by the EWPT is crucial to know whether the LWSM could be tested or not in the next experiments, in particular at the LHC.

With these motivations we have performed an analysis of the EW observables in the LWSM. As in the original formulation [1], we have assumed the principle of minimal flavor violation (MFV) to simplify the flavor structure of the model. The most stringent constraints come from the S and T Peskin-Takeuchi parameters [23]. We present our results as lower bounds on different combinations of the LW-masses of the gauge and quark fields. If we assume degenerate LW-masses for all the fields, the LW-scale allowed by the EWPT is of order 5 TeV and there is little chance to test this model at the LHC. Relaxing the constraint on equal masses, it is possible to find configurations in the parameter space where one of the masses can be as low as $2.4 - 3.5$ TeV, at the price of rising the other masses to be $\gtrsim 5 - 8$ TeV. This situation is more favorable from the experimental side and it might be accessible at the LHC.

Concerning the fine-tuning of the model, a heavy Higgs gives further contributions to S and T pointing in the same direction as the contributions from the LW-fields, and for this reason is strongly disfavoured. Thus, in order to obtain a light Higgs one has to cancel the rather large contributions to the Higgs mass from the LW-particles running in the loops, that are proportional to the LW-masses squared. We estimate the needed tuning of the model to be at least of order a few per cent.

A possible way to relax the constraints from the EWPT would be to generate an extra positive contribution to T without increasing at the same time the S parameter. By including a fourth generation of fermions of SM-type, with their corresponding LW-partners, it is possible to generate a large T, without generating a too large S. To obtain a T parameter of the needed size one has to assume an approximate custodial symmetry for the Yukawas of the fourth generation. We show that for vector LW-masses of order $3\,\mathrm{TeV}$ and Yukawa masses of the fourth generation in the range $0.2 - 1.2\,\mathrm{TeV}$, it is possible to have all the fermionic LW-masses in the range $0.4 - 1.5\,\mathrm{TeV}$ and pass the EWPT.

The paper is organized as follows. In section 2 we give a very brief description of the LWSM, in sections 3 and 4 we compute the tree and radiative contributions to the EW precision parameters. In section 5 we scan over the parameter space of the model and present a detailed analysis of the allowed regions. We consider the extension of the LWSM by including a fourth generation in section 6. We conclude in section 7 and show the details of some of the calculations in the appendices.

2. The LWSM

The LWSM was originally formulated introducing a higher derivative term for each of the SM-fields. The theory contains one new parameter for every SM-field, the LW-mass corresponding to the dimensional coefficient of the associated higher derivative term. The authors of ref. [1] introduced new LW-fields and showed that it is possible to reformulate the theory in terms of these fields in such a way that there are no higher derivative terms in the Lagrangian. In this formulation the LW-masses are the masses of the LW-fields and, although the LW-fields mix with the SM ones, the particle content of the theory is more transparent. The LW-fields have the same quantum numbers as their SM partners and the couplings between the SM and LW-fields are the same as the SM couplings, although the signs of the interactions are not always the usual ones. It is possible to consider even higher derivative terms (*e.g.* six-derivative terms) that will in general lead to additional LW-states, however we will not consider this case. We refer the reader to ref. [1] for the details and quote here some specific interaction terms that are useful to understand the contributions to the EW observables. We will denote the fields associated to the LW-states with a tilde.

The quadratic Lagrangian for the gauge SM and LW-fields, after setting the Higgs to its vacuum expectation value (VEV), is:

$$\mathcal{L}_{2g} = -\frac{1}{2}\mathrm{tr}\left(B_{\mu\nu}B^{\mu\nu} - \tilde{B}_{\mu\nu}\tilde{B}^{\mu\nu} + W_{\mu\nu}W^{\mu\nu} - \tilde{W}_{\mu\nu}\tilde{W}^{\mu\nu}\right)$$
$$-\frac{1}{2}(M_1^2\tilde{B}_\mu\tilde{B}^\mu + M_2^2\tilde{W}_\mu^a\tilde{W}_a^\mu) + \frac{g_2^2 v^2}{8}(W_\mu^{1,2} + \tilde{W}_\mu^{1,2})(W_{1,2}^\mu + \tilde{W}_{1,2}^\mu)$$
$$+\frac{v^2}{8}(g_1 B_\mu + g_1 \tilde{B}_\mu - g_2 W_\mu^3 - g_2 \tilde{W}_\mu^3)(g_1 B^\mu + g_1 \tilde{B}^\mu - g_2 W_3^\mu - g_2 \tilde{W}_3^\mu), \quad (2.1)$$

where $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu, W_\mu = W_\mu^a T^a$ and similar for the other fields, and $g_{1,2}$ are the hypercharge and weak couplings. The sign of the kinetic and mass terms of the LW-fields

are opposite to the usual ones. This sign is responsible for the cancellation of the quadratic divergences as well as the microscopic causality violations by the LW-particles.

The quadratic Lagrangian for the fermionic fields after setting the Higgs to its VEV is:

$$\mathcal{L}_{2\psi} = \sum_{\psi=q_L,u_R,d_R} \bar{\psi} i \not{\partial} \psi - \sum_{\psi=q,u,d} \bar{\tilde{\psi}} (i \not{\partial} - M_\psi) \tilde{\psi}$$
$$-m_u(\bar{u}_R - \bar{\tilde{u}}_R)(q_L^u - \tilde{q}_L^u) - m_d(\bar{d}_R - \bar{\tilde{d}}_R)(q_L^d - \tilde{q}_L^d) + \text{h.c.}, \qquad (2.2)$$

where a generation index is understood, $q^t = (q^u, q^d)$ denotes the $SU(2)_L$ doublet, $m_{u,d} = \lambda_{u,d} v/\sqrt{2}$ and for simplicity we have omitted the leptonic sector. Different to the SM chiral fermions, the LW-fermions combine into Dirac spinors of masses $M_{q,u,d}$. We will assume that the LW-fermions transforming in the same representation of the gauge symmetries have the same mass, this is compatible with the MFV principle [24]. Then the matrices M_ψ of eq. (2.2) are proportional to the identity and we will trade $M_\psi \to \mathbb{1} M_\psi$, with M_ψ a scalar parameter. For effects on FCNC when MFV is not satisfied see ref. [16].

The quadratic Lagrangian for the Higgs field is:

$$\mathcal{L}_{2H} = (\partial_\mu H)^\dagger (\partial^\mu H) - (\partial_\mu \tilde{H})^\dagger (\partial^\mu \tilde{H}) + M_h^2 \tilde{H}^\dagger \tilde{H} - \frac{m_h^2}{2}(h - \tilde{h})^2 \,, \qquad (2.3)$$

where $H^t = (h^+, \frac{v+h+iP}{\sqrt{2}})$ and $\tilde{H}^t = (\tilde{h}^+, \frac{\tilde{h}+i\tilde{P}}{\sqrt{2}})$, $m_h^2 = \lambda v^2/2$ and M_h is the LW-mass. Only the physical Higgs field h and its LW-partner $\tilde{h}$ mix.

The gauge fermionic interactions are:

$$\mathcal{L}_{\text{int}} = - \sum_{\psi=q_L,u_R,d_R} [g_1 \bar{\psi}(\not{B} + \not{\tilde{B}})\psi + g_2 \bar{\psi}(\not{W} + \not{\tilde{W}})\psi]$$
$$+ \sum_{\psi=q,u,d} \left[g_1 \bar{\tilde{\psi}}(\not{B} + \not{\tilde{B}})\tilde{\psi} + g_2 \bar{\tilde{\psi}}(\not{W} + \not{\tilde{W}})\tilde{\psi} \right]. \qquad (2.4)$$

Note that the LW-fermions couple to the gauge fields with the opposite sign compared with the SM-fermions.

For LW-mass scales much larger than the top mass the mixings between the light SM-fermions and the LW-fermions can be neglected. For this reason only the third generation will contribute to the EW precision parameters. In appendix A we diagonalize the fermionic mass matrix. In eqs. (A.5-A.9) we show the physical masses and the matrices connecting the flavor and mass basis for $M_q \neq M_{u,d}$. The case for $M_q = M_{u,d}$ has been considered in ref. [20].

Finally we want to comment on the naturalness of the model. The authors of ref. [1] showed explicitly that the gauge one loop contributions to the Higgs mass are only logarithmically sensitive to the cut-off of the theory. We want to stress that this contribution is proportional to the square of the vector LW mass, M_g, $\delta m_h^2 \simeq \frac{3C_2(N)g^2}{16\pi^2} M_g^2 \log \frac{\Lambda^2}{M_g^2}$, in such a way that the quadratic divergence is recovered when the LW-mass is divergent. The same effect is present in the fermionic contribution to the Higgs mass, $\delta m_h^2 \simeq \frac{N_c \lambda^2}{8\pi^2} M_f^2 \log \frac{\Lambda^2}{M_f^2}$, with M_f the fermionic LW-mass. Therefore, to have a light Higgs in a natural way, the LW-vectors (fermions) should be lighter than $\sim 2\,\text{TeV}$ ($\sim 600\,\text{GeV}$), with a mild dependence on the cut-off Λ.

3. Tree level contributions to the EW precision parameters

We discuss in this section the tree level contributions to the EW precision parameters. We will show that the only parameters that are important in the LWSM are the oblique parameters S and T. In the next section we will compute the 1-loop corrections to S and T and show that the radiative contributions can be as large as the tree level ones.

In the LWSM the mixings between the gauge bosons and their LW partners induce non-canonical couplings for the SM fermions.[1] A shift in the gauge fermion couplings can be reabsorbed into the oblique parameters. Therefore, to correctly define the oblique parameters S, T, U it is necessary to properly normalize the couplings between the fermions and the gauge bosons.[2] We find it useful to work in the effective theory obtained after integrating out the heavy LW fields at tree level. Setting the Higgs field to its VEV, the interactions in the effective theory that are important to normalize the gauge fermion couplings are:

$$
\begin{aligned}
\mathcal{L}_{\text{eff}} = & -g_2 W^{\mu\,1} J_\mu^1 \left[1 - \frac{g_2^2 v^2}{g_2^2 v^2 - 4M_2^2} \right] - g_2 W^{\mu\,2} J_\mu^2 \left[1 - \frac{g_2^2 v^2}{g_2^2 v^2 - 4M_2^2} \right] \\
& - J_\mu^3 \left[g_2 W^{\mu\,3} - (g_2 W^{\mu\,3} - g_1 B^\mu) \frac{g_2^2 v^2 M_1^2}{g_1^2 v^2 M_2^2 + (g_2^2 v^2 - 4M_2^2) M_1^2} \right] \\
& - J_\mu^Y \left[g_1 B^\mu - (g_1 B^\mu - g_2 W^{\mu\,3}) \frac{g_1^2 v^2 M_2^2}{g_1^2 v^2 M_2^2 + (g_2^2 v^2 - 4M_2^2) M_1^2} \right]
\end{aligned}
\tag{3.1}
$$

where J_μ^i are the usual currents of SM fermions, and we have considered that the momentum of the LW-vectors is small, $p^2 \ll M_i^2$. Since the coefficients in eq. (3.1) are the same for all the generations, the same redefinition of the gauge fields leads to canonical gauge couplings for all the SM fermions:

$$
\mathcal{L}_{\text{eff}} = -g_2 \sum_{a=1,2,3} W^{\mu\,a} J_\mu^a - g_1 B^\mu J_\mu^Y \ .
\tag{3.2}
$$

The gauge kinetic and mass terms induce contributions to the oblique parameters after the gauge field redefinitions. The tree level contributions to S and T are:

$$
S = 4\pi v^2 \left(\frac{1}{M_1^2} + \frac{1}{M_2^2} \right) + \mathcal{O}\left(\frac{v^4}{M_i^4} \right) \ ,
\tag{3.3}
$$

$$
T = \pi \frac{g_1^2 + g_2^2}{g_2^2} \frac{v^2}{M_1^2} \ .
\tag{3.4}
$$

Eq. (3.4) is valid to all order in v in the tree level approximation. Moreover, notice that the sign difference between eq. (3.4) and the result of ref. [1] is due to the additional contribution coming from the redefinition of the gauge fields mentioned above. We can see that for $M_1 \to \infty$ the tree level T parameter cancels, as expected since in this limit we partially recover a custodial symmetry in the LW-gauge sector.

---

[1]See ref. [25] for a discussion of this effect in another context.

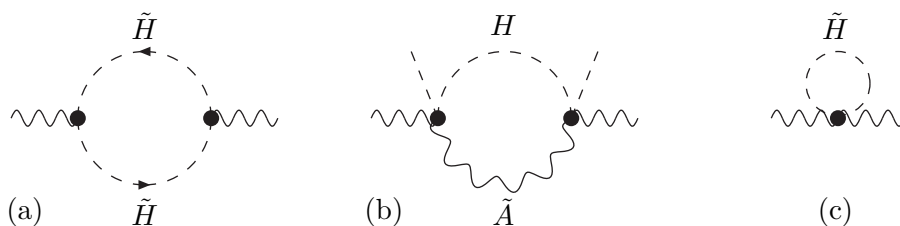[2]This observation was overlooked in ref. [1].

Figure 1: One-loop Feynman diagrams contributing to the oblique parameters involving the Higgs sector.

The U parameter is of order $\mathcal{O}\left(\frac{v^4}{M_i^4}\right)$ and will be neglected in our analysis.

The effective Lagrangian also includes four fermion operators generated by exchange of LW vector fields, with coefficients of order $g_i^2/(2M_i^2)$. The constraints from these operators are weaker than the constraints from the oblique parameters.

The mixings of SM and LW fermions also induce non-canonical couplings, this effect could be important for the b-quark. The mixings between b_L and the LW fermions are of order m_b/M_q. Therefore to protect the $Zb_L\bar{b}_L$ coupling that is in agreement with the SM prediction at the 0.25%, it is enough to have a LW mass $M_q \geq \mathcal{O}(100\text{GeV})$. On the other hand, the experimental measurements of the forward-backward asymmetry of the bottom quark indicate a deviation in the $Zb_R\bar{b}_R$ coupling of order 25%, $\delta g_R^b \sim 0.02$. Since in the LWSM the b_R mixings are of order m_b/M_d, to generate the required anomalous coupling at tree level we would need a very low mass $M_d \sim \mathcal{O}(10\,\text{GeV})$, already excluded.[3]

4. Radiative contributions to S and T

In this section we compute the one-loop contributions to the oblique parameters S and T. The most important contributions come from the third generation of LW-fermions.

4.1 Contributions to T from the gauge-Higgs sector

The one-loop Feynman diagrams involving the LW-Higgs field $\tilde{H}$ are shown in figure 1. We discuss first the contributions to T. There is no custodial symmetry in the LWSM protecting the T parameter. Thus there is no reason to expect finite radiative contributions to T. As expected from the general arguments of Ref [1] there are no quadratic divergences, however, we obtain corrections from the LW-Higgs sector that are logarithmically sensitive to the UV cut-off of the theory.

We consider the different diagrams of figure 1 in some detail. Since the charged and pseudoscalar LW-Higgs fields do not mix with their SM partners, the diagrams (a) and (c) of figure 1 cancel in the combination $\Pi_{11} - \Pi_{33}$ and do not contribute to T.

The diagrams of figure 1(b) with one SM-Higgs and one LW-gauge field can be explicitly computed. For $M_{1,2} \gg m_W^2$ we can perform an expansion in Higgs-VEV insertions. The

---

[3]To agree with the experimental data a $\delta g_R^b \sim 0.17$ is also possible, but it would require even lighter new particles.

leading contribution comes from the zeroth order term, *i.e.*: we neglect the mixings of the LW-gauge fields due to the mass insertions. In this limit the Feynman diagrams give $\Pi_{11}(0) - \Pi_{33}(0) \simeq \frac{g_1^2 g_2^2 v^2}{64\pi^2} \frac{m_h^2}{M_1^2} \log \frac{\Lambda^2}{m_h^2}$. A brief explanation of this result is as follows: there is a factor $g_1 g_2 v/2$ for each vertex, the factor $1/(16\pi^2)$ comes from the loop and the sign is different from the SM-Higgs contribution because the LW propagator has an extra minus sign. Again, this contribution to T cancels for infinite M_1.

From the previous result we obtain $T \simeq \frac{g_1^2 + g_2^2}{4\pi} \frac{m_h^2}{M_1^2} \log \frac{\Lambda^2}{m_h^2}$, that is logarithmically divergent with the cut-off. However, for a light Higgs and LW-gauge masses larger than $\sim 2\,\mathrm{TeV}$, these contributions are smaller than the tree level ones, eq. (3.4), even in the limit of $\Lambda \sim M_{Pl}$. As we will show in the next section, they are also smaller than the fermionic contributions.

There are contributions to T from the diagram of figure 1(c), replacing the LW-Higgs propagator by a LW-gauge one. At leading order in a mass insertion expansion, this contribution exactly cancels because the $SU(2)_L$ LW-gauge fields are degenerate. There is a non-vanishing contribution at second order but it is suppressed by a factor m_W^2/M_2^2, and can be neglected.

4.2 Contributions to S from the gauge-Higgs sector

We discuss the LW-Higgs sector contributions to S from the figure 1. For $m_h \ll M_h$ all the LW-Higgs components are degenerate, thus at leading order in a mass insertion expansion the Feynman diagrams corresponding to figure 1(a) cancel out. The first non-trivial contribution is due to the splitting between the neutral LW-Higgs and the other LW-Higgs components. This gives a small $S \simeq \frac{m_h^2}{24\pi M_h^2}$.

The Feynman diagram of figure 1(b) gives a small contribution also, $S \simeq \frac{1}{2\pi}\left(\frac{m_W^2}{M_2^2} + \frac{m_Z^2 s^2}{M_1^2}\right)$, with $s = \sin\theta_W$.

This contributions to S can be neglected compared with the tree level one, eq. (3.3).

4.3 Fermionic contributions to T

The T parameter measures the amount of isospin breaking, thus the third generation, having the largest Yukawa couplings, gives the dominant contribution compared with the other fermions. We will show that the fermionic contributions of figure 2 dominate also over the other loop corrections. To check our calculations we have computed them in two different ways. We refer the reader to the appendices for the details.

One way to compute the fermionic T is by working in the diagonal mass basis. Inserting the rotation matrices $S_{L,R}^{u,d}$, defined in eqs. (A.8) and (A.9), into the gauge fermion interactions, eq. (2.4), we obtain the couplings between the fermions and the SM-gauge fields in the mass basis. Since there are no mixings in this basis, we just have to sum over all the possible fermionic combinations in the loop diagram of figure 2(a). The matrices $S_{L,R}^{u,d}$ have been calculated in a perturbative mass insertion expansion, then the results obtained by this method are valid for $m_{u,d} \ll M_{u,d,q}$. To obtain a non vanishing T one has to consider at least four mass insertions, this implies that we have to expand $S_{L,R}^{u,d}$ to that
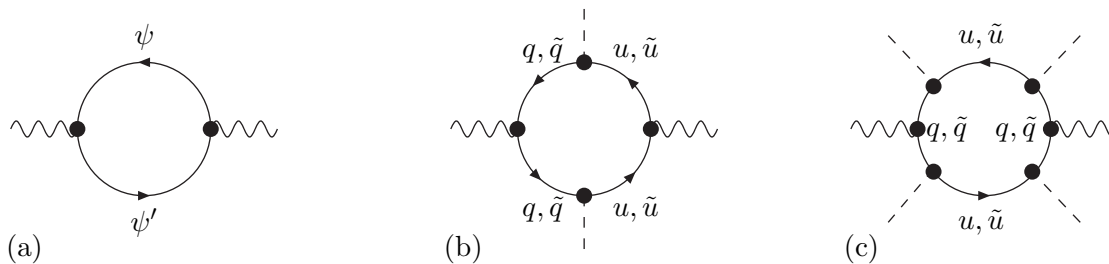
Figure 2: One-loop Feynman diagrams contributing to the oblique parameters involving the fermionic sector. Diagram (a) is the fermionic loop in the mass basis. Diagrams (b,c) are the first non-trivial contributions from the up sector expanding in Yukawa mass insertions.

order. The result is almost independent on M_d because the down Yukawa is much smaller than the up Yukawa for the third generation.

For small LW-fermionic masses ($M_{q,u} \gtrsim m_t$) the convergence of the mass insertion series is rather slow. In fact, for masses $M_{q,u} \lesssim 1\,\text{TeV}$ we have checked that the first non-trivial contribution in the perturbative expansion has large deviations from the non-perturbative one, and can not be trusted. For this reason we will consider also the resummation of the mass insertion series. The diagram of figure 2(c) gives the first non-trivial contribution to T in the flavor basis in the mass insertion expansion. For Π_{33}, the fermionic propagators (at zeroth order in mass insertions) attached to the gauge vertex could be either q_L or $\tilde{q}$, and the fermionic propagators between the Yukawas could be u_R or $\tilde{u}$ (d_R or $\tilde{d}$) for the up (down) contribution. There is a similar diagram for Π_{11}. Using the results of appendices B and C it is possible to obtain the fermionic vacuum polarizations to all orders in the mass insertions. The result for $\Pi_{11}(0) - \Pi_{33}(0)$ is:

$$
\Pi_{11}(0) - \Pi_{33}(0) = \frac{g_2^2 N_c}{4} \int \frac{d^4 p}{(4\pi)^2} \left[\frac{p^6 - 4p^4 M_q^2 + p^2 M_q^4}{(p^2 - M_q^2)^4} + \frac{1}{p^2} - 2\frac{p^2 - 2M_q^2}{(p^2 - M_q^2)^2} \right] \times
$$
$$
\times \left[\frac{m_u^2 M_u^2 (p^2 - M_q^2)}{p^4 (M_u^2 - p^2) + M_q^2 (m_u^2 M_u^2 + p^2 (p^2 - M_u^2))} \right.
$$
$$
\left. - \frac{m_d^2 M_d^2 (p^2 - M_q^2)}{p^4 (M_d^2 - p^2) + M_q^2 (m_d^2 M_d^2 + p^2 (p^2 - M_d^2))} \right]^2 \tag{4.1}
$$

where $m_{u,d}$ stand for the masses of the third generation. Eq. (4.1) includes the contribution from the SM-fermions, that must be subtracted to obtain T. This term is obtained by taking the limit of infinite LW-masses.

The resulting T parameter is negative and it increases for small LW-masses. We make an analysis of the results and its consequences for the LHC in section 5.

4.4 Fermionic contributions to S

Perturbatively the fermionic S parameter counts the number of active fermions in the EW sector. However, at one loop, doublets (N, E) of chiral fermions contribute with $S \sim 1/(6\pi)[1 - 2Y \log(m_N^2/m_E^2)]$, whereas for vector-like fermions the constant term is absent,

$S \sim 2Y/(3\pi) \log(m_N^2/m_E^2)$. The LW-fermions are vector-like and the isospin splitting within a doublet is due to the mixings with the SM fields through the Yukawa couplings. Therefore, for $M_{q,u} \gg m_t$ we expect the LW-fermions to induce a small S at loop level. However, due to the mixings between the SM and LW-fermions, the contributions to S are not so simple, and for $M_{q,u} \sim m_t$ the isospin splitting could be large.

The fermionic one loop Feynman diagrams contributing to S are shown in figure 2. We have computed them using the two methods of section 4.3, by working in the diagonal mass basis and also resumming the mass insertions in the flavor basis. For $M_{q,u} \gtrsim 1.5 \, \text{TeV}$ the exact one-loop calculation computed in the flavor basis and the perturbative calculation in the mass basis agree very well. However, for $M_{q,u} \lesssim 1 \, \text{TeV}$ the perturbative result has large deviations from the full result. We have checked that including higher order terms in the mass insertion expansion the convergence is improved for low values of $M_{q,u}$. We will use the vacuum polarization resumming the mass insertions in our analysis (an expression similar to eq. (4.1), but much longer, can be obtained also in this case, however we omit it for the sake of brevity). For some details on this calculations see the appendix C. The important result is that the fermionic S is negative and small compared with the tree level S of eq. (3.3).

5. Analysis of the EWPT

We make a numerical analysis of the LWSM by scanning over the parameter space of the model: M_q, M_u, M_1, M_2. As we argue in section 4.3 the dependence on M_d is negligible (we have also checked by a numerical calculation that this is true), thus from now on we fix $M_d = 1 \, \text{TeV}$. As argued in sections 4.1 and 4.2, our results are almost independent on m_h and M_h provided that the Higgs is light, $m_h \sim 114 \, \text{GeV}$, and the LW-Higgs is sensibly heavier than the SM-Higgs. In any case, as we will show, a heavy SM-Higgs is strongly disfavored by the EWPT.

To obtain a better understanding of the importance of the S and T parameters in constraining the model we show in figure 3 the 68% and 95% Confidence Level contours in the (S, T) plane, as obtained from the LEP Electroweak Working Group [26], together with the LWSM predictions for several values of the LW-masses. It is clear from figure 3 that there is no region in the parameter space lying within the 68% Confidence Level contour (we have considered LW-masses not larger than 10 TeV). There is however a small but sizable region of the 95% Confidence Level contour that is covered. Choosing all the LW-masses to be equal corresponds to the large yellow points in figure 3. In this case only masses above 5 TeV enter into the allowed region. The other coloured points in figure 3 correspond to one of the LW-masses being light (lighter than 4 TeV), whereas the black dots are for all the LW masses being heavier than 4 TeV. We can see that most of the configurations with light new particles do not satisfy the EWPT, whereas most of the configurations that pass the EWPT do not have any light new particle.

It is also evident from figure 3 that most of the configurations have a too large positive S and negative T parameters. The positive S contribution is mainly generated at tree level by the non-canonical fermionic gauge couplings, see eq. (3.3). The T parameter has a
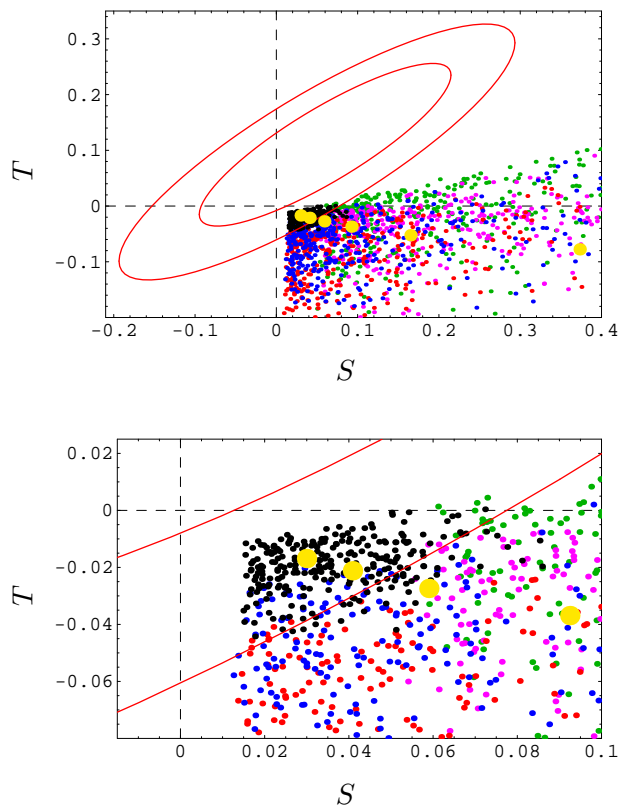
Figure 3: 68% and 95% Confidence Level contours in the (S, T) plane, and LWSM predictions. The black dots indicate points where all four masses M_1 , M_2, M_q , M_u are larger than 4 TeV. Coloured points correspond to cases where at least one mass is less than 4 TeV. The colour indicates which mass is below 4 TeV: green, magenta, red, blue dots correspond to M_1 , M_2 , M_q , M_u less than 4 TeV, respectively. The yellow dots correspond to taking all masses equal and $7, 6, 5, 4 \ldots$ TeV, from left to right.

tree level positive contribution, eq. (3.4), and a negative one-loop contribution due to the third generation of LW-fermions. For light LW-fermions the one-loop fermionic correction dominates over the tree level one resulting in a negative T. A light LW-vector could cancel the large negative T generated by the fermions, but generating at the same time a too large S.

We quote now the minimum values of LW-masses that pass the EWPT. The lightest M_1 (M_2) lying inside the 95% contour is $M_1 \simeq 3.2$ TeV (3.8 TeV), and corresponds to a green (magenta) point in figure 3. For a green (magenta) point to lie inside the ellipse, M_2, M_q, M_u (M_1, M_q, M_u) have to be heavier than $\sim 5.4, 5.2, 3.6$ TeV ($\sim 7.0, 8.1, 6.0$ TeV), respectively. The lightest M_q (M_u) inside the ellipse is $M_q \simeq 3.5$ TeV ($M_u \simeq 2.4$ TeV). For a red (blue) point to lie inside the ellipse, M_1, M_2, M_u (M_1, M_2, M_q) have to be heavier than $\sim 7.5, 7.0, 4.9$ TeV ($\sim 3.9, 4.7, 4.2$ TeV), respectively.

A heavy SM-Higgs gives an extra negative T and a positive S [23]. Thus it points in the wrong direction and gives stronger constraints for the LWSM. As shown in sections 4.1 and 4.2, the contributions from the LW-Higgs can not alleviate this situation.
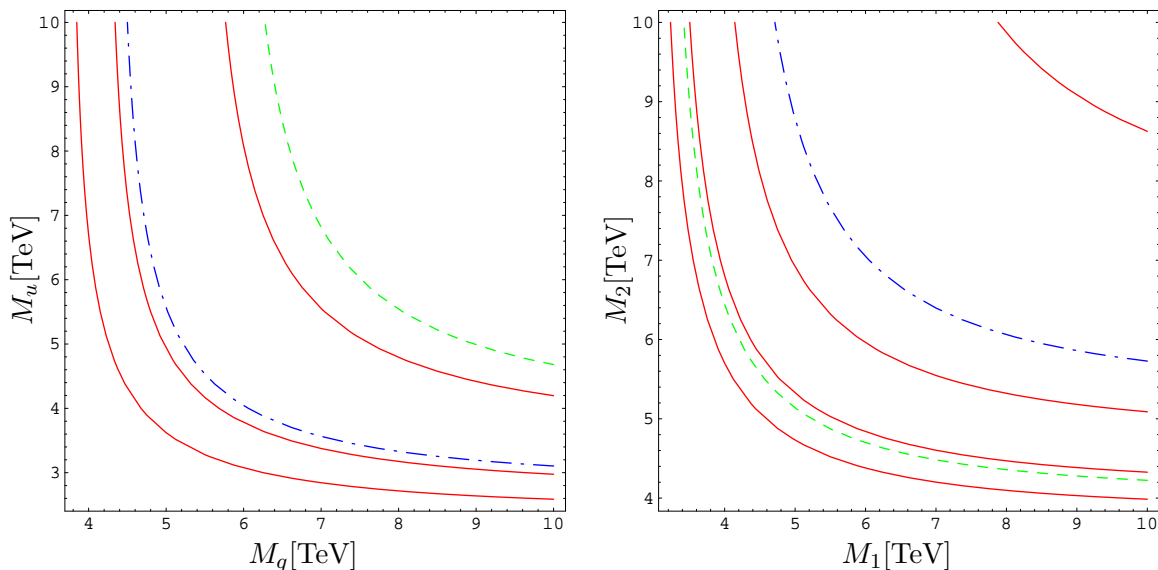
Figure 4: Values of the LW-masses allowed by the EWPT (95% Confidence Level contour). The region above the lines is allowed by the EWPT and the region below the lines is forbidden. On the left we show the plane (M_q, M_u) for fixed values of the LW-vector masses. The red lines correspond (from left to right) to $M_1 = M_2 = 7, 6, 5$ TeV. The dashed green line to $M_1 = 10, M_2 = 4$ TeV, and the dot-dashed blue line to $M_1 = 4, M_2 = 10$ TeV. On the right we show the plane (M_1, M_2), for fixed LW-fermionic masses. The red lines correspond (from left to right) to $M_q = M_u = 7, 6, 5, 4$ TeV. The dashed green line to $M_q = 10$ TeV and $M_u = 4$ TeV, and the dot-dashed blue line to $M_q = 4$ TeV and $M_u = 10$ TeV.

In figure 4 we show the LW-fermionic masses M_q, M_u allowed by the EWPT for fixed values of $M_{1,2}$. We have considered the 95% confidence level constraints on the S, T parameters. The lines divide the parameter space in an upper region allowed by the EWPT and a lower region that does not pass the EWPT.

In order to obtain a rather light LW-fermion, for example an $SU(2)_L$ singlet, $\tilde{u}$, with a mass of order $2.5 - 3$ TeV, we are forced by the EWPT to have heavy LW-vectors and also a heavy LW-fermion doublet, $\tilde{q}$, with masses larger than ~ 5 TeV.

In figure 4 we show also the LW-vector masses M_1, M_2 allowed by the EWPT for fixed values of $M_{q,u}$. The regions above (below) the lines (do not) pass the EWPT. To obtain a light vector the other vector and the fermions are forced to be heavy, with masses larger than $\sim 5 - 6$ TeV. In any case the LW-vector masses can not be lighter than 3 TeV. The lightest vectors are slightly heavier than the lightest fermions, as they give larger contributions to S. In general the LW-vector $\tilde{B}$ can be lighter than $\tilde{W}$. This is because, given a positive S, the EWPT prefer a positive T, that is generated by $\tilde{B}$ and not by $\tilde{W}$.

6. Extending the LWSM with a fourth generation

We consider in this section a very simple extension of the LWSM that can provide positive contributions to T and a rather small S. We add a fourth generation of fermions with the same quantum numbers and chiralities as the ordinary SM generations, together with their
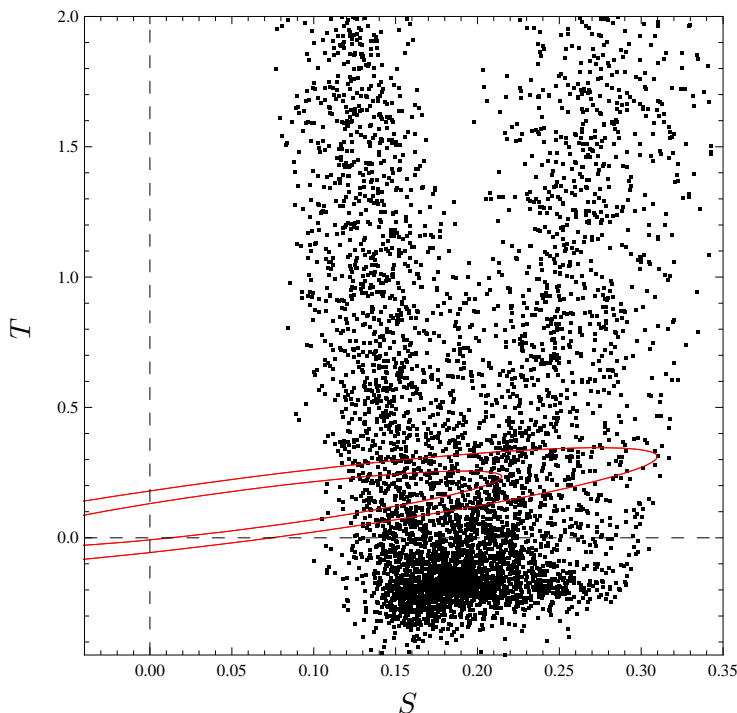
Figure 5: 68% and 95% Confidence Level contours in the (S, T) plane, and predictions of the LWSM with a fourth generation. The vector LW-masses are fixed to $3\,\text{TeV}$ and $M_q \simeq 1.5\,\text{TeV}$. The Yukawas and fermionic LW-masses take values in the range $0.2 - 1.5\,\text{TeV}$.

LW-partners. The important terms in the Lagrangian for the EWPT are still described by eqs. (2.2) and (2.4), with a generation index including the fourth generation. For simplicity we will consider that M_ψ, acting on a space of dimension four, is still proportional to the identity. Therefore, the only new parameters are the Yukawa couplings of the fourth generation. We will consider only the effects of the quarks of the fourth generation, moreover, it is very simple to include the leptons to cancel the anomalies. Ignoring for the moment the mixings between the SM-fermions and the fourth generation, the new physical effects are contained in $m_{u,d}^4 = \lambda_{u,d}^4 \, v/\sqrt{2}$.

As explained in section 4.4, a generation of SM-quarks with a rather small isospin splitting produces a $S \sim 0.1$, whereas vector-like quarks do not produce S in the limit of no isospin splitting. Moreover, for a rather large isospin splitting the S parameter generated by SM-quarks decreases and the S generated by vector-like quarks remains very small, $S \lesssim 0.04$ for $m_N \lesssim 2m_E$. Therefore, taking into account the results for the minimal LWSM, an extra small contribution to S can be consistent with the EWPT if there is also a small and positive contribution to T.

The T parameter generated by new fermions is proportional to the isospin splitting of the new sector. Thus the splittings in the Yukawas of the fourth generation and in the LW-sector produce contributions to T. Since the mass of the down quark of the fourth generation can be large, T has a strong dependence with M_d in this case. The T parameter generated by a fourth generation with their LW-partners, without constraints in the isospin violation, will be in general of order 1, much larger than the needed T. We have checked
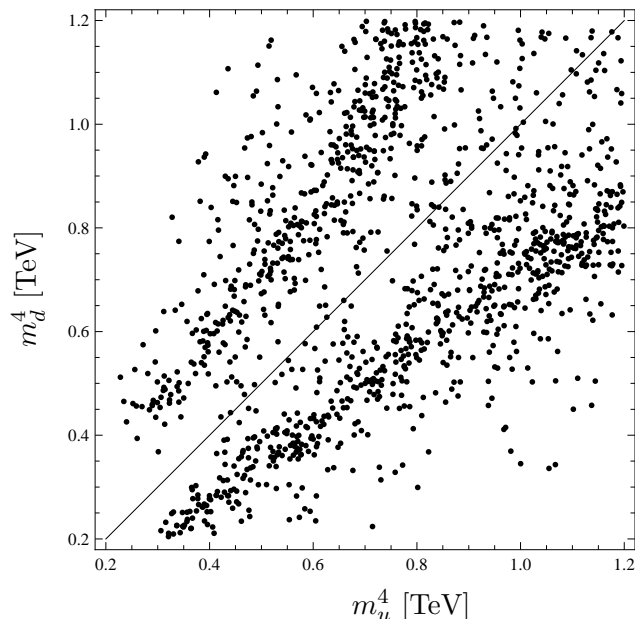
Figure 6: Points in the (m_u^4, m_d^4) plane satisfying the EWPT at the 95% Confidence Level in the LWSM with a fourth generation. The line corresponds to $m_u^4 = m_d^4$.

that this is indeed the situation in the present proposal. Therefore, to generate a positive T of the appropriate size, it is necessary to constrain the isospin splitting.

It is immediate to extend the results of sections 4.3 and 4.4 to include the one loop effects of a fourth generation. We have scanned over the parameter space fixing the vector LW-masses to be of order $\sim 3\,\mathrm{TeV}$, in order to suppress the large tree level contributions to S and T. It is found that a heavy $M_q \sim 3-4\,\mathrm{TeV}$ is preferred by the data, but a lower $M_q \sim 1.5\,\mathrm{TeV}$ is still consistent with the EWPT for light $m_{u,d}^4$ and $M_{u,d}$.

In figure 5 we show the 68% and 95% Confidence Level contours in the (S, T) plane together with the predictions of the LWSM with a fourth generation. We have considered Yukawas and fermionic LW-masses in the range $0.2 - 1.5\,\mathrm{TeV}$. The first thing one can notice is that a much larger region of the ellipses in the (S, T) plane is covered in this case, compared with the figure 3 of the minimal LWSM. Also there is a rather large range of values of T covered, as expected if there is no restriction in the isospin splitting. A heavier $M_q, \sim 3\,\mathrm{TeV}$, increases T, allowing a larger overlap between the dense region and the ellipses.

In figure 6 we show the regions in the plane (m_u^4, m_d^4) preferred by the EWPT. We find that the isospin violation due to the Yukawas of the fourth generation has to be rather small, satisfying the approximate relation $\frac{|m_u^4 - m_d^4|}{m_u^4 + m_d^4} \sim 0.3$. On the other hand, since the effect from the LW-fermions is much smaller, the constraints in the isospin splitting are much weaker for this sector. We find that there are no regions excluded in the (M_u, M_d) plane, provided that M_u and M_d are larger than $\sim 0.4\,\mathrm{TeV}$.

7. Conclusions

We have performed a careful scan over the parameter space of the minimal LWSM and have determined the regions that pass the EWPT. The most stringent constraints come from the S and T Peskin-Takeuchi parameters. In particular, the most important restrictions come from the tree level contributions to S and T and from the one-loop fermionic contribution to T.

We have shown that it is necessary to choose very specific values of the LW-masses to obtain light LW particles that could eventually be discovered at the LHC. The masses of the vectorial LW-particles are always of order 3.2 TeV or heavier, with the lightest value obtained at the price of rising the masses of the fermionic LW-particles to be at least of order $6 - 8$ TeV. The fermionic spectrum of LW-particles can be somewhat lighter than the vectorial one, and it is possible to have fermions with masses as low as 2.4 TeV. This can be done rising the LW-masses of the other fermions and vectors to be at least of order $5 - 8$ TeV. Whether these heavy states could be produced and detected at the LHC deserves a careful study, some analysis has been done in refs. [19, 21].

The only possible exception to the previous bounds may be the down LW-fermion, whose mass is not well constrained. Since the bottom Yukawa is small, the EWPT do not give important restrictions on M_d. Although the model does not explain the origin of the LW-masses, we can expect that the same mechanism gives masses to all the LW-fermions. In this case M_d may be of the same order as the other fermionic masses M_u and M_q.

The degree of tuning of the model depends on the scale Λ where new physics beyond the LWSM shows up. For degenerate LW-masses, and in the most favorable case, with a small $\Lambda \sim \mathcal{O}(10\,\text{TeV})$, we estimate a degree of tuning that is at least of order a few per cent (see the last paragraph of section 2). For larger Λ the tuning becomes of order a few per mille. In the scenario where a little hierarchy in the LW-spectrum is allowed, the degree of fine-tuning increases to order a few per mille already with a small Λ. Thus, although the LWSM can solve the hierarchy problem by cancelling the quadratic divergences of the SM, to pass the EWPT with its minimal version one has to reintroduce some degree of tuning.

The constraints from the EWPT can be relaxed extending the minimal LWSM in such a way that there is an extra positive contribution to T without increasing much, at the same time, the S parameter. We have shown that this can be done including a fourth generation of fermions with its LW-partners. Without any assumption in the isospin violation of the fourth generation Yukawas and in the LW-fermionic sector, the generated T is too large. Our study shows that the effect of isospin violation from the Yukawas is larger than the effect from the LW-fermions. To generate the appropriate T one has to impose an approximate custodial symmetry for the Yukawas. The amount of isospin violation required by the data is of order 30%. We have considered vector LW-masses of order 3 TeV to suppress the tree level S and T, and we have shown that in this case the fermionic LW-masses can be as small as $\sim 0.4 - 1.5$ TeV. Therefore, with this very simple extension it is possible to obtain a LWSM that can be tested at the LHC. A careful study of this scenario and other possible extensions beyond the minimal LWSM is needed.

Acknowledgments

A. Diagonalization of the fermionic mass matrix

We consider the diagonalization of the fermionic mass matrix of the third generation. We collect the up fermions into a three dimensional vector in the following way:

$$\psi_L^{u\,t} = (q_L^u, \tilde{q}_L^u, \tilde{u}_L) \qquad \psi_R^{u\,t} = (u_R, \tilde{u}_R, \tilde{q}_R^u) \,, \tag{A.1}$$

and similarly for the down fermions. We adopt the same basis as [20], but with a different notation. Using eq. (A.1) we can write the quadratic fermionic Lagrangian (2.2) as:

$$\mathcal{L}_{2\psi} = \bar{\psi}^u i\, \partial\!\!\!/ \eta \psi^u - \bar{\psi}_R^u \mathcal{M}_u \eta \psi_L^u - \bar{\psi}_L^u \eta \mathcal{M}_u^\dagger \psi_R^u + \dots \,, \tag{A.2}$$

where the dots stand for the down sector, $\eta = \mathrm{diag}(1, -1, -1)$ and

$$\mathcal{M}_u \eta = \begin{pmatrix} m_u & -m_u & 0 \\ -m_u & m_u & -M_u \\ 0 & -M_q & 0 \end{pmatrix} \tag{A.3}$$

The mass matrix $\mathcal{M}_u$ can be diagonalized by independent left and right symplectic rotations $S_{L,R}$ satisfying:

$$\mathcal{M}_{u,\mathrm{phys}} \eta = S_R^\dagger \mathcal{M}_u \eta S_L \,, \qquad S_R \eta S_R^\dagger = \eta \,, \qquad S_L \eta S_L^\dagger = \eta \,, \tag{A.4}$$

where $\mathcal{M}_{u,\mathrm{phys}}$ is the physical mass matrix, which is diagonal.

To obtain explicit analytic expressions we expand the solutions in powers of Yukawa insertions $m_{u,d}$. Thus our results are well approximated by the first terms in this expansion in the limit $\epsilon_{q,u} = \frac{m_u}{M_{q,u}} \ll 1$. For the elements of the diagonal matrix $\mathcal{M}_{u,\mathrm{phys}}$ we obtain the following:

$$m_u \left[1 + \frac{1}{2}(\epsilon_q^2 + \epsilon_u^2) + \frac{1}{8}(7\epsilon_q^4 + 7\epsilon_u^4 + 10\epsilon_q^2 \epsilon_u^2) \right] + \mathcal{O}(\epsilon_{q,u}^6) \,, \tag{A.5}$$

$$M_u \left[1 - \frac{\epsilon_u^2}{2} \frac{M_q^2}{M_q^2 - M_u^2} - \frac{\epsilon_u^4}{8} \frac{5M_q^6 - 9M_q^4 M_u^2}{(M_q^2 - M_u^2)^3} \right] + \mathcal{O}(\epsilon_{q,u}^6) \,, \tag{A.6}$$

$$M_q \left[1 + \frac{\epsilon_q^2}{2} \frac{M_u^2}{M_q^2 - M_u^2} + \frac{\epsilon_q^4}{8} \frac{5M_u^6 - 9M_u^4 M_q^2}{(M_q^2 - M_u^2)^3} \right] + \mathcal{O}(\epsilon_{q,u}^6) \,. \tag{A.7}$$

For the matrices $S_{L,R}$ we obtain

$$S_L - 1 = \tag{A.8}$$

$$\begin{bmatrix}
\frac{\epsilon_u^2}{2} + \frac{4\,\epsilon_q^4 + 8\,\epsilon_q^2\,\epsilon_u^2 + 11\,\epsilon_u^4}{8} & -\frac{\left(\epsilon_q^8\left(4\,\epsilon_q^2 - 7\,\epsilon_u^2\right)\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^3} + \frac{\epsilon_q^4}{-\epsilon_q^2+\epsilon_u^2} & -\epsilon_u - \frac{\epsilon_u^5\left(-4\,\epsilon_q^2 + 3\,\epsilon_u^2\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^2} \\
-\epsilon_q{}^2 - \frac{\epsilon_q^2\left(4\,\epsilon_q^2 + 3\,\epsilon_u^2\right)}{2} & -\frac{\left(\epsilon_q^4\,\epsilon_u^2\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^2} + \frac{\epsilon_u^8\left(4\,\epsilon_q^4 - 16\,\epsilon_q^2\,\epsilon_u^2 + 23\,\epsilon_u^4\right)}{8\left(\epsilon_q^2-\epsilon_u^2\right)^4} & \frac{\epsilon_q^2\,\epsilon_u\left(-2\,\epsilon_q^4 + 4\,\epsilon_q^2\,\epsilon_u^2 - 2\,\epsilon_u^4\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^3} + \frac{\epsilon_q^2\,\epsilon_u\left(4\,\epsilon_q^2\,\epsilon_u^4 - \epsilon_u^6\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^3} \\
-\epsilon_u - \frac{\epsilon_u\left(2\,\epsilon_q^2 + 3\,\epsilon_u^2\right)}{2} & \frac{\epsilon_u^6\,\epsilon_u\left(2\,\epsilon_q^2 - 5\,\epsilon_u^2\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^3} + \frac{\epsilon_q^2\,\epsilon_u}{\epsilon_q^2-\epsilon_u^2} & \frac{\epsilon_u^4\left(-2\,\epsilon_q^2 + \epsilon_u^2\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^2} + \frac{\epsilon_u^8\left(36\,\epsilon_q^4 - 36\,\epsilon_q^2\,\epsilon_u^2 + 11\,\epsilon_u^4\right)}{8\left(\epsilon_q^2-\epsilon_u^2\right)^4}
\end{bmatrix}$$

$$S_R - 1 = \tag{A.9}$$

$$\begin{bmatrix}
\frac{\epsilon_q^2}{2} + \frac{11\,\epsilon_q^4 + 8\,\epsilon_q^2\,\epsilon_u^2 + 4\,\epsilon_u^4}{8} & -\epsilon_q - \frac{\epsilon_q^5\left(3\,\epsilon_q^2 - 4\,\epsilon_u^2\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^2} & \frac{\epsilon_u^4}{\epsilon_q^2-\epsilon_u^2} + \frac{\epsilon_u^8\left(-7\,\epsilon_q^2 + 4\,\epsilon_u^2\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^3} \\
-\epsilon_u{}^2 - \frac{\epsilon_u^2\left(3\,\epsilon_q^2 + 4\,\epsilon_u^2\right)}{2} & \frac{\epsilon_q\,\epsilon_u^2\left(\epsilon_q^6 - 4\,\epsilon_u^4\,\epsilon_q^2\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^3} + \frac{\epsilon_q\,\epsilon_u^2\left(2\,\epsilon_q^4 - 4\,\epsilon_q^2\,\epsilon_u^2 + 2\,\epsilon_u^4\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^3} & \frac{-\left(\epsilon_q^2\,\epsilon_u^4\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^2} + \frac{\epsilon_u^8\left(23\,\epsilon_q^4 - 16\,\epsilon_q^2\,\epsilon_u^2 + 4\,\epsilon_u^4\right)}{8\left(\epsilon_q^2-\epsilon_u^2\right)^4} \\
-\epsilon_q - \frac{\epsilon_q\left(3\,\epsilon_q^2 + 2\,\epsilon_u^2\right)}{2} & \frac{\epsilon_q^4\left(\epsilon_q^2 - 2\,\epsilon_u^2\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^2} + \frac{\epsilon_q^8\left(11\,\epsilon_q^4 - 36\,\epsilon_q^2\,\epsilon_u^2 + 36\,\epsilon_u^4\right)}{8\left(\epsilon_q^2-\epsilon_u^2\right)^4} & \frac{\epsilon_q\,\epsilon_u^6\left(5\,\epsilon_q^2 - 2\,\epsilon_u^2\right)}{2\left(\epsilon_q^2-\epsilon_u^2\right)^3} - \frac{\epsilon_q\,\epsilon_u^2}{\epsilon_q^2-\epsilon_u^2}
\end{bmatrix}$$

The solution for the down-sector can be obtained from the up-sector simply by changing $u \to d$.

The authors of ref. [20] considered the case $M_q = M_u$. Their solutions can not be obtained from the case $M_q \neq M_u$ that is singular in the limit $M_q \to M_u$. There is a singularity because in that limit there is a degenerate eigenspace of dimension two, with no preferred eigenvectors.

B. Mass insertion resummation of the flavor propagators

In this appendix we provide expressions for the flavor propagators to all orders in the mass insertions. Due to the mixings between SM and LW-fermions, the fermionic propagators are also mixed in the flavor basis in the mass insertion expansion. The resummation of the mass insertion series can be performed and the flavor propagators shown in eq. (B.7) are those used in sections 4.3 and 4.4 to compute the full radiative fermionic contributions to T and S respectively. We illustrate the method for resumming the mass insertion series with a particular example. Other cases are simple variations of the one discussed below and they can be obtained by using the same procedure.

Consider the case of the resummed propagator $(\tilde{S}_{q^u})$ of a SM up-fermion in a $SU(2)_L$ doublet. The first three terms of the series are shown in figure 7. The first term corresponds to the zeroth order propagator (S_q) which is obtained from the SM kinetic term in the Lagrangian given by eq. (2.2):

$$\mathcal{L}_{2\psi} \supset \bar{q}_L i\,\slashed{\partial} q_L \,, \tag{B.1}$$

and it takes the form:

$$\tilde{S}_{q^u}^{(0)} \equiv S_q = P_L \frac{1}{\slashed{p}} \,, \tag{B.2}$$

where $P_L = (1 - \gamma^5)/2$. The following two terms in the expansion, containing at least two

Figure 7: Expansion in mass insertions of the propagator ($\tilde{S}_{q^u}$) of a SM up-fermion in the SU(2)$_L$ doublet. S_q stands for the q^u propagator with no mass insertions. Besides, $u-\tilde{u}$ ($q-\tilde{q}$) corresponds to internal zeroth order propagators for u_R (q^u) and $\tilde{u}_R$ ($\tilde{q}^u$).

mass insertions, can be derived by taking into account the mixing mass term:

$$\mathcal{L}_{2\psi} \supset m_u(\bar{u}_R - \bar{\tilde{u}}_R)(q_L^u - \tilde{q}_L^u) \, . \tag{B.3}$$

The first and second order propagators in mass insertions are found to be respectively:

$$\tilde{S}_{q^u}^{(1)} = m_u^2 P_L S_q (S + S_{\tilde{u}}) P_L S_q = m_u^2 S_q \, \slashed{p} P_L S_q A_u \, , \tag{B.4}$$

$$\tilde{S}_{q^u}^{(2)} = m_u^2 P_L S_q (S + S_{\tilde{u}}) m_u (S_q + S_{\tilde{q}}) m_u (S + S_{\tilde{u}}) P_L S_q = m_u^2 S_q \, \slashed{p} P_L S_q A_u (m_u^2 p^2 A_q A_u) \, ,$$

where A_u and A_q, and the zeroth order propagators for u_R, $\tilde{u}_R$ and $\tilde{q}$ (S, $S_{\tilde{u}}$ and $S_{\tilde{q}}$ respectively) are given by:

$$A_u \equiv \frac{1}{p^2} - \frac{1}{p^2 - M_u^2} \, , \qquad A_q \equiv \frac{1}{p^2} - \frac{1}{p^2 - M_q^2} \, ,$$

$$S \equiv P_R \frac{1}{\slashed{p}} \, , \qquad S_{\tilde{u}} \equiv -\frac{1}{\slashed{p} + M_u} \, , \qquad S_{\tilde{q}} \equiv -\frac{1}{\slashed{p} + M_q} \, , \tag{B.5}$$

with $P_R = (1+\gamma^5)/2$. Notice the extra minus sign and the absence of any chirality projector in $S_{\tilde{q}}$ and $S_{\tilde{u}}$. From the results in eq. (B.4), it is not difficult to infer the generic n-th term and the sum of the series can be obtained:

$$\tilde{S}_{q^u} = S_q + m_u^2 S_q \, \slashed{p} P_L S_q A_u \sum_{n=0}^{\infty} (m_u^2 p^2 A_q A_u)^n = S_q + \frac{m_u^2 S_q \, \slashed{p} P_L S_q A_u}{1 - m_u^2 p^2 A_q A_u} \, . \tag{B.6}$$

Note that $\tilde{S}_{q^u}$ has only an even number of mass insertions.

Following similar arguments, it is possible to obtain all the resummed flavor propagators. For SM and LW fermions of the third generation, charged under SU(2)$_L$, there are four different types of propagators arising from an even number of mass insertions. Initial and final legs carry the same SM ($\tilde{S}_q$) or LW-fermion ($\tilde{S}_{\tilde{q}}$), or they are attached to different fermions: the initial leg is a SM ($\tilde{S}_{q\tilde{q}}$) or a LW-fermion ($\tilde{S}_{\tilde{q}q}$) –the subscript q ($\tilde{q}$) stands for up or down SM (LW) fermions in the SU(2)$_L$ doublet. This class of propagators enters the calculation of both S and T parameters. The computation of the vacuum polarization contributions to S also requires propagators with an odd number of mass insertions that can be obtained with the same method outlined above. There are four relevant types of them according to all possible combinations of SM and LW fermions in the SU(2)$_L$ doublet and up or down singlets coupled to hypercharge ($\tilde{M}_{q(u,d)}, \tilde{M}_{q(\tilde{u},\tilde{d})}, \tilde{M}_{\tilde{q}(\tilde{u},\tilde{d})}$ and $\tilde{M}_{\tilde{q}(u,d)}$).

Those obtained by interchanging initial and final legs ($\tilde{M}_{ij} \to \tilde{M}_{ji}$) are needed as well. Resumming all possible insertions of mixing mass terms, we obtain the following expressions for propagators in the up-sector:

$$\tilde{S}_{q^u}(p) = S_q + \frac{m_u^2 S_q \not{p} P_L S_q A_u}{1 - m_u^2 p^2 A_q A_u} \quad , \qquad \tilde{S}_{\tilde{q}^u}(p) = S_{\tilde{q}} + \frac{m_u^2 S_{\tilde{q}} \not{p} P_L S_{\tilde{q}} A_u}{1 - m_u^2 p^2 A_q A_u} \quad ,$$

$$\tilde{S}_{q^u \tilde{q}^u}(p) = -\frac{m_u^2 S_q \not{p} P_L S_{\tilde{q}} A_u}{1 - m_u^2 p^2 A_q A_u} \quad , \qquad \tilde{S}_{\tilde{q}^u q^u}(p) = -\frac{m_u^2 S_{\tilde{q}} \not{p} P_L S_q A_u}{1 - m_u^2 p^2 A_q A_u} \quad ,$$

$$\tilde{M}_{q^u u}(p) = -\frac{m_u S_q P_R S}{1 - m_u^2 p^2 A_q A_u} \quad , \qquad \tilde{M}_{q^u \tilde{u}}(p) = \frac{m_u S_q P_R S_{\tilde{u}}}{1 - m_u^2 p^2 A_q A_u} \quad ,$$

$$\tilde{M}_{\tilde{q}^u u}(p) = \frac{m_u S_{\tilde{q}} P_R S}{1 - m_u^2 p^2 A_q A_u} \quad , \qquad \tilde{M}_{q^u \tilde{u}}(p) = -\frac{m_u S_{\tilde{q}} P_R S_{\tilde{u}}}{1 - m_u^2 p^2 A_q A_u} \quad , \qquad \text{(B.7)}$$

where S_q, S, $S_{\tilde{u}}$, $S_{\tilde{q}}$, A_u and A_q have been defined in eqs. (B.2) and (B.5). $\tilde{M}_{ji}$ has a similar expression to $\tilde{M}_{ij}$, only the place of the zeroth order propagators must be interchanged as in the case of $\tilde{S}_{q^u \tilde{q}^u}$ and $\tilde{S}_{\tilde{q}^u q^u}$. Note that the series for $\tilde{M}_{ij}$ and $\tilde{S}_{ij}$ start from one and two mass insertions respectively since the kinetic terms are flavor diagonal.

The fermionic spectrum of the up-sector is given by the poles of $\tilde{S}_{q^u}$.

Flavor propagators for the down-sector are obtained from those above by changing $u \to d$ in eqs. (B.7).

C. Fermionic contribution to the vacuum polarization

We show in this appendix the fermionic contributions to the vacuum polarization associated to the S and T parameters:

$$S = \frac{16\pi}{g_1 g_2} \Pi'_{3B}(0) \ , \qquad T = \frac{4\pi}{g_2^2 s^2 c^2 m_Z^2} [\Pi_{11}(0) - \Pi_{33}(0)] \ , \qquad \text{(C.1)}$$

with $\Pi_{\mu\nu} = g_{\mu\nu} \Pi + (q_\mu q_\nu \text{terms})$.

We consider first the perturbative expansion of Π_{33} from figure 2(c). Since u_R, d_R and their LW-partners are singlets of $SU(2)_L$, and the gauge interactions do not mix SM and LW-fermions, the fermionic legs attached to one of the gauge vertices correspond either to q or to $\tilde{q}$. However, it is possible to have q-legs attached to one of the vertices and either q or $\tilde{q}$-legs attached to the other vertex, and similar for $\tilde{q}$. Using the propagators of appendix B we can write the up contribution to Π_{33} to all orders in the mass insertion expansion as:

$$\Pi_{33}^{\mu\nu} = -\frac{g_2^2}{4} \text{tr} \int \frac{d^4 p}{(2\pi)^4} (\gamma^\mu \tilde{S}_{q^u} \gamma^\nu \tilde{S}_{q^u} + \gamma^\mu \tilde{S}_{\tilde{q}^u} \gamma^\nu \tilde{S}_{\tilde{q}^u} - 2\gamma^\mu \tilde{S}_{q^u \tilde{q}^u} \gamma^\nu \tilde{S}_{\tilde{q}^u q^u}), \qquad \text{(C.2)}$$

and a similar contribution from the down sector. The minus sign in the last term is because the LW-fermions couple to the SM-gauge fields with a sign flip compared with the SM-fermions, see eq. (2.4).

The contributions to Π_{11} can be obtained in a similar way, and is given by:

$$\Pi_{11}^{\mu\nu} = -\frac{g_2^2}{2} \text{tr} \int \frac{d^4 p}{(2\pi)^4} (\gamma^\mu \tilde{S}_{q^u} \gamma^\nu \tilde{S}_{q^d} + \gamma^\mu \tilde{S}_{\tilde{q}^u} \gamma^\nu \tilde{S}_{\tilde{q}^d} - 2\gamma^\mu \tilde{S}_{q^u \tilde{q}^u} \gamma^\nu \tilde{S}_{\tilde{q}^d q^d}). \qquad \text{(C.3)}$$

The S parameter is proportional to $\Pi'_{3B}(0)$. The fermionic contribution to Π_{3B} is more involved because the fermions u_R, d_R and their LW-partners couple to hypercharge. Therefore we have to consider the diagrams of figures 2(b) and (c). The contribution from figure 2(c) is similar to Π_{33}, with the appropriate charges:

$$\Pi^{\mu\nu}_{3B} = -\frac{g_1 g_2}{12}\mathrm{tr}\int \frac{d^4p}{(2\pi)^4}(\gamma^\mu \tilde{S}_{q^u}\gamma^\nu \tilde{S}_{q^u} + \gamma^\mu \tilde{S}_{\tilde{q}^u}\gamma^\nu \tilde{S}_{\tilde{q}^u} - 2\gamma^\mu \tilde{S}_{q^u\tilde{q}^u}\gamma^\nu \tilde{S}_{\tilde{q}^u q^u}), \qquad \text{(C.4)}$$

and a similar contribution from the down sector with a minus sign due to the different weak charge.

Figure 2(b) gives contributions with $q, \tilde{q}$-legs attached to W_3 and the singlets $u_R, d_R, \tilde{u}, \tilde{d}$ attached to B. Using the propagators of appendix B, the up contribution to Π_{3B}, to all orders in the mass insertion expansion, is:

$$\Pi^{\mu\nu}_{3B} = -\frac{g_1 g_2}{3}\mathrm{tr}\int \frac{d^4p}{(2\pi)^4} \times \qquad \text{(C.5)}$$
$$\times (\gamma^\mu \tilde{M}_{q^u u}\gamma^\nu \tilde{M}_{u q^u} - \gamma^\mu \tilde{M}_{q^u \tilde{u}}\gamma^\nu \tilde{M}_{\tilde{u} q^u} - \gamma^\mu \tilde{M}_{\tilde{q}^u u}\gamma^\nu \tilde{M}_{u \tilde{q}^u} + \gamma^\mu \tilde{M}_{\tilde{q}^u \tilde{u}}\gamma^\nu \tilde{M}_{\tilde{u} \tilde{q}^u}).$$

The contribution from the down sector can be obtained from eq. (C.5) by changing the factor $\frac{1}{3}$ by $\frac{1}{6}$ and changing the indices $u \to d$.

References

[1] B. Grinstein, D. O'Connell and M.B. Wise, *The Lee-Wick standard model*, *Phys. Rev.* **D 77** (2008) 025012 [arXiv:0704.1845].

[2] T.D. Lee and G.C. Wick, *Negative metric and the unitarity of the S matrix*, *Nucl. Phys.* **B 9** (1969) 209.

[3] T.D. Lee and G.C. Wick, *Finite theory of quantum electrodynamics*, *Phys. Rev.* **D 2** (1970) 1033.

[4] B. Grinstein, D. O'Connell and M.B. Wise, *Massive vector scattering in Lee-Wick gauge theory*, *Phys. Rev.* **D 77** (2008) 065010 [arXiv:0710.5528].

[5] R.E. Cutkosky, P.V. Landshoff, D.I. Olive and J.C. Polkinghorne, *A non-analytic S matrix*, *Nucl. Phys.* **B 12** (1969) 281.

[6] N. Nakanishi, *Lorentz noninvariance of the complex-ghost relativistic field theory*, *Phys. Rev.* **D 3** (1971) 811.

[7] T.D. Lee and G.C. Wick, *Questions of Lorentz invariance in field theories with indefinite metric*, *Phys. Rev.* **D 3** (1971) 1046.

[8] N. Nakanishi, *Remarks on the dipole-ghost scattering states*, *Phys. Rev.* **D 3** (1971) 1343.

[9] I. Antoniadis and E.T. Tomboulis, *Gauge invariance and unitarity in higher derivative quantum gravity*, *Phys. Rev.* **D 33** (1986) 2756.

[10] D.G. Boulware and D.J. Gross, *Lee-Wick indefinite metric quantization: a functional integral approach*, *Nucl. Phys.* **B 233** (1984) 1.

[11] K. Jansen, J. Kuti and C. Liu, *The Higgs model with a complex ghost pair*, *Phys. Lett.* **B 309** (1993) 119 [hep-lat/9305003].

[12] K. Jansen, J. Kuti and C. Liu, *Strongly interacting Higgs sector in the minimal standard model?*, *Phys. Lett.* **B 309** (1993) 127 [`hep-lat/9305004`].

[13] Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. Schroeder, *New Higgs physics from the lattice*, `PoS(LATTICE 2007)056` [`arXiv:0710.3151`].

[14] F. Knechtli, N. Irges and M. Luz, *New Higgs mechanism from the lattice*, `arXiv:0711.2931`.

[15] B. Grinstein and D. O'Connell, *One-loop renormalization of Lee-Wick gauge theory*, `arXiv:0801.4034`.

[16] T.R. Dulaney and M.B. Wise, *Flavor changing neutral currents in the Lee-Wick standard model*, *Phys. Lett.* **B 658** (2008) 230 [`arXiv:0708.0567`].

[17] F. Wu and M. Zhong, *The Lee-Wick fields out of gravity*, *Phys. Lett.* **B 659** (2008) 694 [`arXiv:0705.3287`].

[18] J.R. Espinosa, B. Grinstein, D. O'Connell and M.B. Wise, *Neutrino masses in the Lee-Wick standard model*, `arXiv:0705.1188`.

[19] T.G. Rizzo, *Searching for Lee-Wick gauge bosons at the LHC*, *JHEP* **06** (2007) 070 [`arXiv:0704.3458`].

[20] F. Krauss, T.E.J. Underwood and R. Zwicky, *The process $gg \to h_0 \to \gamma\gamma$ in the Lee-Wick standard model*, *Phys. Rev.* **D 77** (2008) 015012 [`arXiv:0709.4054`].

[21] T.G. Rizzo, *Unique Identification of Lee-Wick gauge bosons at linear colliders*, *JHEP* **01** (2008) 042 [`arXiv:0712.1791`].

[22] R. Barbieri and A. Strumia, *The 'LEP paradox'*, `hep-ph/0007265`.

[23] M.E. Peskin and T. Takeuchi, *A New constraint on a strongly interacting Higgs sector*, *Phys. Rev. Lett.* **65** (1990) 964; *Estimation of oblique electroweak corrections*, *Phys. Rev.* **D 46** (1992) 381.

[24] R.S. Chivukula and H. Georgi, *Composite technicolor standard model*, *Phys. Lett.* **B 188** (1987) 99;
G. D'Ambrosio, G.F. Giudice, G. Isidori and A. Strumia, *Minimal flavour violation: an effective field theory approach*, *Nucl. Phys.* **B 645** (2002) 155 [`hep-ph/0207036`].

[25] K. Agashe, A. Delgado, M.J. May and R. Sundrum, *RS1, custodial isospin and precision tests*, *JHEP* **08** (2003) 050 [`hep-ph/0308036`].

[26] LEP collaboration, J. Alcaraz et al., *Precision electroweak measurements and constraints on the standard model*, `arXiv:0712.0929`.